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# Macroscopic open strings and gravitational confinement

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## Abstract

We consider classical solutions for strings ending on magnetically charged black holes in four-dimensional Kaluza–Klein theory. We examine the classical superstring and the global vortex, which can be viewed as a nonsingular model for the superstring. We show how both of these can end on a Kaluza–Klein monopole in the absence of self-gravity. Including gravitational back-reaction gives rise to a confinement mechanism of the magnetic flux of the black hole along the direction of the string. We discuss the relation of this work to localized solutions in ten-dimensional supergravity.

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**Keywords:** Supergravity solutions; Black holes; Topological defects

## 1. Introduction

The interplay between macroscopic soliton solutions in field theory, and microscopic quantum physics has always been a fruitful and fascinating one. Solitons, while generally ‘heavy’ and ‘classical’ in nature, nonetheless are an important and indeed integral part of the particle spectrum of the theory. The effort to fully integrate these objects into quantum theory has led to a deeper understanding of existing theories. In particular, the study of the ‘solitonic’ D-branes [1] has revolutionized the study of string theory.

There are two different perspectives from which one can view a D-brane; one is an inherently stringy point of view, which is that of applying Dirichlet, rather than Neumann, boundary conditions to open strings [2]. On the other hand, a somewhat more

‘classical’ perspective views the brane as a possibly extended solution of low energy string supergravity carrying Ramond–Ramond (RR) charge [3]. Combining these two approaches has led to many new insights, such as a better understanding of black hole entropy [4] and the AdS/CFT correspondence [5].

There are also classical solutions carrying Neveu–Schwarz charge, the NS5-brane and the string. This latter solution is often viewed as a macroscopic fundamental superstring [6]. Viewing strings and D-branes classically as black brane solutions to supergravity, one can ask a whole range of interesting gravitational questions, such as what happens when two branes meet? There are many known solutions for intersecting branes (see [7] and references therein), however these often ‘de-localized’ in the sense that the solution depends only on the mutually orthogonal directions. A genuinely localized intersection has proved somewhat elusive, although one can construct near-horizon solutions (e.g., [8]), and semi-localized solutions [9],

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indeed, in some cases localization is thought not to be possible [10].

If however this rather nice correspondence between supergravity solutions and the stringy picture is neat and closed, then there surely should exist solutions of fundamental strings terminating on D-branes. This statement has some secondary implications. If it is indeed possible to find a classical solution for a string terminating on a D-brane, then it should, in principle, be possible for a macroscopic fundamental string to split, in a manner similar to the four-dimensional cosmic string [11,12], since it can nucleate a D/anti-D-brane pair along its length. This could have potential implications for the dynamics of a superstring network in the early universe, where additional interactions of string splitting and rejoining would also have to be taken into account.

To explore this question we consider a rather simpler one. Rather than examining the ten-dimensional problem, for which one might expect that fully localised intersecting brane solutions would be a necessary precursor, we look at two rather simpler toy models: a classical superstring, and its nonsingular field-theoretic cousin—the global vortex—in four-dimensional Kaluza–Klein (KK) gravity. This ‘ministring’ can be viewed either as a truncation of the IIA superstring, or the dimensional reduction of a membrane from five-dimensional gravity [13]. These strings can split by the nucleation of the four-dimensional equivalent of D0- or D6-branes, which are now the electric and magnetic KK black holes [14]. The D0-brane is a gravitational wave travelling around the internal circle, and is singular, however, the ‘D6’-brane is the KK monopole [15], and is constructed from the Taub–NUT instanton, which, at least for unit monopole charge, is completely regular from the five-dimensional point of view.

In what follows we start by taking a rather empirical approach, initially ignoring the gravitational back reaction of the string (working in the limit in which the string is arbitrarily light compared to the monopole) first exhibiting a string worldsheet which satisfies charge conservation in the sense of the NSNS and RR form fields that live in the spacetime. Similarly, for the global string, initially, we solve only the field equations on the KK monopole background, before coupling in gravity. Even this background or ‘probe’

solution already gives important clues about the gravitational backreaction.

## 2. Background solutions

In order to find a configuration corresponding to a classical superstring ending on a monopole, consider its interpretation as a double-dimensional reduction of a supermembrane. We are then looking for a configuration involving a membrane in five dimensions terminating on the core of the KK monopole which is nonsingular from the five-dimensional point of view, therefore the membrane cannot have any ends. Fortunately, because of the Hopf-fibration of the KK monopole, it is easy to see how to wrap the membrane so that it has no ends, yet appears to terminate from the dimensionally reduced point of view: one wraps the membrane around the fifth dimension along the ‘south’ direction of the Taub–NUT instanton, but not along the ‘north’ direction. This is a regular configuration with no boundaries (other than that at infinity) and yet from the four-dimensional point of view looks like a string with an end. This problem has in fact been well studied in the M-theory context [16,17], where there are families of static M2-branes given by holomorphic curves on the (hyper-Kähler) Taub–NUT manifold. These have the lower dimensional interpretation of D2-branes in the background of a D6-brane with fundamental string charge. The configuration we have is a five-dimensional version of the limit of one of the curves presented in [16] in which a D2-brane is dragged past the KK-monopole leaving a fundamental string connecting the core of the monopole to infinity.

This clearly represents a geometric solution to the problem, but can it be consistently coupled in to gravity and the 3-form field that also exists in the toy 5-dimensional gravity? To answer this, consider a nonsingular field-theoretic model of the classical superstring—the global vortex—which is a topological defect solution to the  $U(1)$  theory defined by the Lagrangian

$$\begin{aligned}\mathcal{L} &= |\nabla_\mu \Phi|^2 - \frac{\lambda}{4} (|\Phi|^2 - \eta^2)^2 \\ &= \eta^2 \left\{ (\nabla_\mu X)^2 + X^2 (\nabla_\mu X)^2 - \frac{\lambda \eta^2}{4} (X^2 - 1)^2 \right\}\end{aligned}\tag{2.1}$$

writing  $\Phi = \eta X e^{i\chi}$ . This represents a nonsingular version of the string since the scalar Goldstone boson,  $\chi$ , is canonically conjugate to the Kalb–Ramond two-form,  $B_{\mu\nu}$ , of the string (or the three-form  $C_{\mu\nu\lambda}$  of the membrane) in four (five) dimensions. Suppose we consider this  $U(1)$  theory in five dimensions in the background of the KK monopole:

$$ds^2 = dt^2 - \left(1 + \frac{E}{r}\right) d\mathbf{r}^2 - \left(1 + \frac{E}{r}\right)^{-1} [dx^5 + E(1 - \cos\theta) d\varphi]^2, \quad (2.2)$$

where  $d\mathbf{r}^2$  represents  $\mathbb{R}^3$  in spherical polar coordinates  $(r, \theta, \varphi)$ . In order to obtain a vortex ending on the KK monopole, we must choose the phase,  $\chi$ , to be nonsingular along one of the polar axes. It is not difficult to see that the choice  $\chi = x^5/2E$  satisfies this requirement. Along the North polar axis,  $\Phi$  is regular without requiring  $X = 0$ , whereas along the South polar axis the metric (2.2) has a coordinate singularity, and we must transform to coordinates which are nonsingular in the southern hemisphere:  $\tilde{x}^5 = x^5 + 2E\varphi$ , which is the analogue of the Wu–Yang gauge patching for the Dirac monopole [18]. Hence  $\chi = \tilde{x}^5/2E - \varphi$ , in the vicinity of this axis, which corresponds to a vortex with winding number  $-1$ .

Having decided there is a sensible choice for the phase of the scalar field, one might reasonably ask whether this corresponds to a genuine solution for the vortex. This necessitates solving the  $X$ -equation

$$\begin{aligned} & \frac{[r^2 X_{,r}]_{,r}}{r^2} + \frac{[\sin\theta X_{,\theta}]_{,\theta}}{r^2 \sin\theta} \\ &= \frac{1}{2} \left(1 + \frac{E}{r}\right) X(X^2 - 1) \\ &+ \frac{X}{4} \left[ \frac{1}{E^2} \left(1 + \frac{E}{r}\right)^2 + \frac{\tan^2(\theta/2)}{r^2} \right] = 0, \quad (2.3) \end{aligned}$$

where we have chosen to set the vortex width  $1/\sqrt{\lambda} \times \eta = 1$ . Fortunately, in the limit  $E \gg 1$ , this can be approximately solved analytically, using an analogue of the thin-string approximation used in [19] to obtain solutions for vortices in a black hole background, by noting that if we set

$$X = X(R) = X(2\sqrt{r(r+E)} \cos(\theta/2)),$$

then the  $X$ -equation reduces to

$$\begin{aligned} & -X'' \left[ 1 + \frac{R^2(3r+2E)}{4(r+E)^3} \right] - \frac{X'}{R} \left[ 1 + \frac{R^2(5r+6E)}{8(r+E)^3} \right] \\ &+ \frac{X^2}{R^2} \left[ 1 + \frac{R^2(r+2E)}{4E^2(r+E)} \right] + \frac{1}{2} X(X^2 - 1) \\ &= 0 \end{aligned} \quad (2.4)$$

which is simply the equation for the scalar field of a global vortex in flat space with  $O(E^{-2})$  corrections. Note however, that unlike the flat space global vortex, these corrections mean that this vortex does not tend asymptotically to the vacuum, but rather  $X \rightarrow 1 - 1/E^2$ . Obviously this is just an analytic approximation, and one can integrate the equations numerically, two illustrations of which are shown in Fig. 1. The plots represent the contours of the  $X$ -field, ( $X = 0.1, 0.2, \dots$ ) and show how the vortex spreads as  $E$  is reduced. Below  $E \simeq 1.7$  the  $X = 0.9$  contour ceases to exist, and the ripple is a manifestation of its marginal nature.

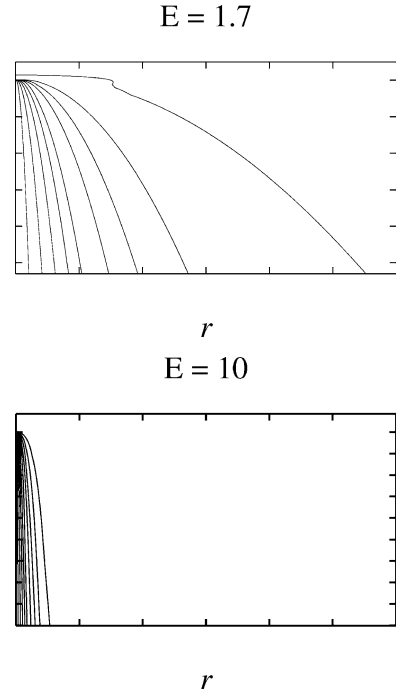


Fig. 1.  $X$ -contour lines in the monopole background. The plots are in “real space” with the  $\varphi$  direction suppressed. The ripple in the  $X = 0.9$  contour on the first plot occurs because that contour is marginal and about to disappear as  $E$  drops in value.

Having used the global vortex as an illustration, one can now read off the form of the superstring, since in five dimensions a 3-form field is dual to a scalar field, therefore we simply dualise the solution for  $\chi$  in the global vortex to find the four form field strength appropriate to the membrane:

$$F_4 = Q \left[ E \sin \theta \left( 1 + \frac{r}{E} \right)^2 dt \wedge dr \wedge d\theta \wedge d\varphi + \tan(\theta/2) dt \wedge dr \wedge d\theta \right] \wedge [dx^5 + E(1 - \cos \theta) d\varphi]. \quad (2.5)$$

When reduced to four spacetime dimensions, this gives rise to the fields

$$H = \tan(\theta/2) dt \wedge dr \wedge d\theta, \quad (2.6)$$

$$\tilde{F}_4 = E \sin \theta \left( 1 + \frac{r}{E} \right)^2 dt \wedge dr \wedge d\theta \wedge d\varphi, \quad (2.7)$$

where  $H = dB$  is the ‘NSNS’ two form field strength, and  $\tilde{F}_4 = F_4 + A \wedge H$  is the ‘RR’ four form (with  $A$  the RR 1-form).

Notice how  $|\tilde{F}_4|^2$  actually tends to a constant at infinity. The reason for this is that in order for the fields to wrap around the internal dimension in such a way that they close off at the north pole in a regular fashion, yet still correspond to a string at the south pole, they must wind around the internal direction, the size of which tends to a constant at infinity, therefore the part of the field which winds around that direction will contribute a constant amount. Alternatively, from the four-dimensional point of view, the interaction of the  $H$ -field generated by the string with the  $A$ -field coming from the KK reduction via the Chern–Simons term causes a long range effect if we wish the fields to conspire to make the configuration regular on the north pole, but have a source on the south pole. In other words, bulk terms matter for classical superstrings.

In each case the superstring and the global vortex can be painted on to the KK monopole background in order to give a configuration corresponding to a terminating string, however the field configuration has a nonvanishing local energy density at infinity. This will have significant implications for the gravitational back reaction of such configurations which we will now discuss.

### 3. Gravitational back reaction

Once we include the gravitational back reaction of the string, we can no longer assume that the background will be similar to the KK-monopole. Indeed, there are two very good reasons for supposing that the spacetime will be radically different from the monopole, and therefore that a conventional linearized analysis will not apply. The first is the presence of these bulk terms—the long range fields that are present in order to allow the string to end. The other reason is the asymmetry of the configuration. Usually, when one linearizes around a background configuration one assumes that when the source is switched on, the spacetime is perturbed in some localised sense. Here however, we are adding a string along one polar axis of the monopole. In the case of a standard local cosmic string ending on a black hole, the solution is altered from the static Schwarzschild solution to the  $C$ -metric [20], which although static, contains acceleration horizons at large radius. In both ways of thinking, the addition of the string has had a long range effect.

In order to investigate the gravitational back reaction, instead of performing a perturbation analysis, we will consider a more general metric

$$ds^2 = e^{2\phi/\sqrt{3}} g_{\mu\nu} dx^\mu dx^\nu - e^{-4\phi/\sqrt{3}} [dx^5 + A_\mu dx^\mu]^2. \quad (3.1)$$

Of course, this metric will not be entirely general, the four-dimensional component will be axisymmetric and static, and the electromagnetic potential will be appropriate to a magnetic solution,  $A_\mu = A \partial_\mu \varphi$ , such that  $A = 0$  along the north axis of symmetry, which will, as before, be the axis pointing away from the terminating string.

To look for a membrane solution in the five-dimensional metric which is nonsingular on this axis, we try  $F_4 = \star 2Q dx^5$ . Substituting the metric (3.1) in the five-dimensional action and integrating out over  $x^5$  yields the four-dimensional action

$$S = \int d^4x \sqrt{g} \left[ -R + 2(\nabla\phi)^2 - \frac{1}{4} e^{-2\sqrt{3}\phi} F^2 + 2Q^2 A_\mu^2 - 2Q^2 e^{2\sqrt{3}\phi} \right], \quad (3.2)$$

where the factors of  $e^{2\phi/\sqrt{3}}$  in (3.1) have been chosen to put the four-dimensional action in the so-called

Einstein frame, in which the gravitational part of the action appears in the Einstein–Hilbert form. In addition to the usual KK terms, this action also contains a mass term for the KK  $U(1)$ -field, and a ‘cosmological’ Liouville potential for the dilaton. This action (3.2), in the absence of the  $Q^2 A_\mu^2$  term, has been analysed with the result that there is a “no go” theorem for spherically symmetric black holes [21]—there are no spherically symmetric black hole solutions which are asymptotically flat or de Sitter. However, this action and our situation has two crucial differences, the presence of the mass term for the gauge field, and the lack of spherical symmetry—a vortex terminating on a black hole is manifestly not spherically symmetric.

An obvious candidate for a nonspherically symmetric metric is of course the dilatonic  $C$ -metric of Dowker et al. [22]. This consists of a black hole under uniform acceleration generated by a conical deficit extending towards infinity and is a solution for the action (3.2) above with  $Q^2 = 0$ . The key features of the geometry are this conical singularity and an acceleration horizon generated by the acceleration of the black hole. For  $RA \ll 1$ , the conical singularity has a deficit angle of order  $RA$ , and the acceleration horizon is at a radius of order  $A^{-1}$ . Meanwhile, the effects of the Liouville potential will become relevant at a scale of order  $Q^{-1}$ , therefore, the dilatonic  $C$ -metric can only be appropriate as an asymptotic solution if the acceleration horizon occurs well before the Liouville and electromagnetic mass terms are relevant: i.e., if  $A^{-1} \ll Q^{-1}$ .

In order to explore this question in greater detail, consider the global vortex. In Einstein gravity, the global vortex spacetime is slightly subtle, since in order to have a regular spacetime one strictly needs to introduce a time dependence or negative cosmological constant [23]. Nonetheless, in order to explore the relative importance of the various terms we will look for a global vortex solution by maintaining the form of the Goldstone field  $\chi = x^5/R$  (where  $2\pi R$  is the periodicity of  $x^5$ ), then integrating out over  $x^5$  gives the four-dimensional effective action

$$S_{\text{eff}} = \int \sqrt{g} \left[ -R + 2(\nabla\phi)^2 - \frac{1}{4}e^{-2\sqrt{3}\phi}F^2 - \eta^2 \frac{X^2}{R^2}e^{2\sqrt{3}\phi} + \eta^2 \left( (\nabla X)^2 + \frac{X^2}{R^2}A_\mu^2 \right. \right.$$

$$\left. - \frac{\lambda\eta^2}{4}e^{2\phi/\sqrt{3}}(X^2 - 1)^2 \right]. \quad (3.3)$$

The first three terms are the standard KK gravity terms. The last three terms (grouped together with the factor of  $\eta^2$ ) come from the kinetic and potential terms of the  $\phi$ -field. However, the kinetic term of the Goldstone  $\chi$ -field gives two contributions (as seen in the  $Q^2$  terms of (3.2)), one of which is the gauge field term and the other the Liouville term of  $X^2 e^{2\sqrt{3}\phi}$ . The reason for putting this term separately is to emphasize the grouping of the  $\eta^2$ -terms. These are very close to the Lagrangian terms for a local  $U(1)$ -vortex, indeed, if we identify  $1/R$  with  $e$ , then apart from the Liouville term, this is precisely the Lagrangian of a dilatonic local vortex, with Bogomolnyi parameter  $\beta = 1/2e^2\eta^2 = R^2/2\eta^2$ . (Note that this is reminiscent of the KK vortices of Dvali et al. [24], however, they broke the  $U(1)$   $\partial_5$  Killing symmetry with a braneworld, whereas we have not.) The gravitational interactions of such vortices were studied in [25], where it was shown that they could be used to smooth out conical deficits in standard ‘vacuum’ spacetimes including those with dilatonic black holes.

Briefly, in [25] it was shown that a dilatonic local vortex, like the standard cosmic string, has a conical deficit in the vicinity of the core, which smooths out the apex of the cone. However, the main difference between dilatonic and Einstein strings is that, depending on how the local vortex couples to the dilaton, it is now possible for spacetime to be nonasymptotically locally flat far from the core, with the dilaton providing the curvature. Only those vortices which couple canonically to the dilaton (i.e., in the string frame if we have string gravity, or the KK frame for KK gravity) avoid this fate. In general in Einstein gravity, it is always possible to use a local cosmic string to smooth out the conical deficits present in a metric [12], for example, the  $C$ -metric which represents two accelerating black holes being pulled away from each other by conical deficits extending to infinity. In dilaton gravity, it was shown in [25] that a dilaton vortex threading a black hole would, if noncanonically coupled, add dilaton charge to the black hole and asymptote the vortex metric. However, for the dilatonic  $C$ -metrics, the conical deficit could only be smoothed if the coupling of the vortex to

the dilaton was canonical. Fortunately, the coupling in (3.3) is precisely canonical.

Now, the analytic analysis in [25] was undertaken supposing  $\beta \simeq O(1)$ , whereas we have  $\beta = R^2/2\eta^2$ , with  $R \gg 1$  (large KK monopole mass) and in addition  $\eta \ll 1$  (as  $\eta^2$  really stands for  $8\pi G\eta^2$  in vortex units, and we are expanding around the low string mass limit). Therefore, we are in a relatively little explored parameter space of the local vortex (some work was done on large  $\beta$  vortices interacting with nondilatonic black holes in [26]). However, by combining results and intuition from local vortices with black holes, as well as the global vortex itself, we can in the absence of the ‘Liouville’ term describe the spacetime.

There will be three main regions:  $r \ll R$ ,  $R \ll r \ll R\eta^{-1}$ , and  $r \gg R\eta^{-1}$ , where  $r$  is to be understood in a qualitative sense as representing the distance scale on which one is examining the spacetime. Roughly speaking these three scales represent the scale on which spacetime is inherently five-dimensional in nature; where the solution is four-dimensional in nature but the Liouville term is not yet relevant; and finally the long range effects of all the terms in the action.

The immediate effect of the vortex core is to provide a snub-nosed conical deficit. This effect is strongly localized around the core of the vortex, and therefore ought not to be particularly affected by the Hopf fibration of the magnetic black hole. There will therefore be a region around the south polar axis which is a conical deficit in the  $\varphi$  polar angle. At intermediate scales, we are well outside the vortex core, and in a four-dimensional régime, yet still well below the scale at which the Liouville and mass terms are relevant. Since the vortex couples in the KK frame, we expect from [25] that the dilatonic metric of [22] will describe the metric at this scale (with the vortex smoothing out the core of the conical deficit). Finally, at large scales the electromagnetic flux confines, and in the absence of the Liouville term we would simply have a vacuum  $C$ -metric.

Such is the spacetime in the absence of the Liouville term. Now let us consider inclusion of this term, which becomes relevant at  $O(R\eta^{-1})$ . First of all, if this scale is outside the acceleration horizon of the  $C$ -metric, then it is clearly irrelevant. The acceleration horizon of the  $C$ -metric is at  $r \sim O(A^{-1})$ , where  $RA \sim \eta^2$  is the deficit angle of the  $C$ -metric.

Thus  $A^{-1} = O(R\eta^{-2}) \gg R\eta^{-1}$  for  $\eta \ll 1$ . Therefore, the Liouville scale is well within the acceleration horizon—a conclusion we expect to be true for the string/membrane as well. For the global vortex, note that it is precisely this Liouville term which is inherently a global vortex term. If we were simply looking for a five-dimensional global vortex membrane, then we could always consider reducing over the angular variable which represented the winding of the Goldstone field. This would give rise to precisely that Liouville term. For a simple cylindrically symmetric vortex in the absence of the electromagnetic field, it is this term which is responsible for the singularity in the static metric for the vortex [27]. However, to render the global vortex nonsingular, one can add a negative cosmological constant, or worldbrane curvature [23], the latter of which causes a compactification of spacetime which becomes a small deformation of the de Sitter hyperboloid [28].

Applying these results to the case at hand, one would therefore expect that the unchecked Liouville term could well render the acceleration horizon of the  $C$ -metric singular, however, one can remove this singularity by adding a small negative cosmological constant, or time-dependence to the metric. Since adding time-dependence means that the metric will now depend on three variables, a rather difficult problem, we will simply assume that a tiny negative cosmological constant has been added to cancel the Liouville term in an analogous way to the self-gravitating global vortex, and we are left with a vacuum  $C$ -metric with a minutely small acceleration.

#### 4. Discussion

So, overall the geometry has three different approximate descriptions: first, the near-field, or core, which for the global string is a snub-nosed cone—for the superstring, this will simply be its local near-core metric. Second there will be a mid-field approximation in which the geometry and dilaton will have the form of the dilatonic  $C$ -metric. Finally, on the large scale, the mass term for the electromagnetic field will cause that flux to confine as well, and (with the proviso of a checking-term in either the action or the intrinsic worldbrane geometry) we will have a standard  $C$ -metric.

We can ask how these conclusions are altered if we try to explore a different range of parameter space, either by lowering the compactification radius  $R$ , or increasing the gravitational strength of the global vortex  $\eta^2$ . The plots in Fig. 1 show that for the global vortex at least, we can lower  $R$  to a similar order of magnitude as the vortex width before we run into trouble. Similarly, analytic arguments on the existence of vortex solutions in [28] show that we can raise  $\eta^2$  to  $O(1)$  without destroying the general conclusions discussed here. We therefore expect that these qualitative results hold true even for strings ending on monopoles with a similar mass.

Clearly this is only a toy model, and cannot directly be used to draw any conclusions for the ten-dimensional problem, however, there are some interesting features which crop up here that might have some analogue in the higher-dimensional case. One is the presence of the bulk terms giving the cosmological term in the action. This was a feature of the finite size of the compactification radius in five dimensions. While we might expect this effect to be ameliorated in higher dimensions, in that it may not lead to a cosmological term, we might expect that the energy density of the various RR and NSNS fields will not fall-off as rapidly as for the superstring itself. The other curious feature we have seen is the mass term for the RR-field, which causes confinement of the magnetic flux. Again, this effect could be lessened, but it does give an interesting potential picture for the distortion of the flux around the intersection point. In fact, if one plots the magnetic flux of the near horizon solution for the string ending on the monopole one does see some evidence of this.

The other main way in which a higher-dimensional problem will differ is in the number of degrees of freedom of the solution. By working in only four dimensions, the problem reduced to an effectively two-dimensional question: the distance from the monopole core, and the angle from the string. We can view any classical supergravity problem as a dimensional reduction over the ‘inessential’ coordinates (i.e., those that represent symmetries of the metric) down to the space on which the metric depends. For two variables we can always express this space as conformally flat. However, as soon as the string ends on an extended brane, there are three physical variables—the distance along the string from the endpoint, the distance along

the brane from where the string touches, and finally, the mutually orthogonal distance from the system. Unfortunately, we cannot write a three-dimensional problem in a conformally flat, or even necessarily diagonal fashion. This shows up in the perturbative analysis of [29]. Since our universe is however four-dimensional up to fairly high energies, it is reasonable to make a four-dimensional approximation to the problem.

However, it is amusing that the act of ending the string on the black hole causes its flux to confine, and therefore removes all evidence of its charge from the asymptotic observer, and leaves it with only a ‘label’ attached to the end of the string. Whereas Nielsen and Olesen [30] originally used the  $U(1)$  Abelian Higgs model to construct a physical realisation of the Nambu–Goto string; by considering a field theoretic realisation of an open superstring, a flux confinement mechanism switches on, and the string ends up as a truly confined flux tube with (Abelian—as we have a unit charged monopole) ‘quarks’ at each end.

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